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Estimation of Several Turbulent Fluctuation Quantities Using an Approximate Pulsatile Flow Model

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Estimation of Several Turbulent Fluctuation Quantities Using an Approximate Pulsatile Flow Model

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ABSTRACT

Turbulent fluctuation behavior is approximately modeled using a pulsatile flow model analogy.. This model follows as an extension to the turbulent laminar sublayer model developed by Sternberg (1962) to be valid for a fully turbulent flow domain. Here unsteady turbulent behavior is modeled via a sinusoidal pulsatile approach. While the individual modes of the turbulent flow fluctuation behavior are rather crudely modeled, approximate temporal integration yields plausible estimates for Root Mean Square (RMS) velocity fluctuations. RMS pressure fluctuations and spectra are of particular interest and are estimated via the pressure Poisson expression. Both RMS and Power Spectral Density (PSD), i.e. spectra are developed. Comparison with available measurements suggests reasonable agreement. An additional fluctuating quantity, i.e. RMS wall shear fluctuation is also estimated, yielding reasonable agreement with measurement.

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NOMENCLATURE

Symbols

a	Dimensionless model constant
B	Inner aw constant B=5
c	Locally defined constant
C_f	Skin friction $C_f = \frac{2\tau_w}{\rho U^2}$
C_{f_0}	Undisturbed free-stream skin friction
Ci	Cosine integral special function
f	Separation solution function pressure Poisson
GM	Geometric average
I	Turbulence intensity (absolute value)
K	Clauser turbulent viscosity constant
k	Wave number
k_0	dimensionless wave number $k_0 = k\delta$
L	Streamwise length scale
M	Free-stream Mach number
p’	Pressure fluctuation amplitude
R	Correlation function
Re	Reynolds number
Re_x	Streamwise flat plate Reynolds number
Re_θ	Momentum thickness Reynolds number
Si	Sine integral special function
t	time
T	Period time scale
u	Streamwise turbulent mean flow
U	Free stream turbulent mean flow velocity
U’	Unsteady turbulent mean flow velocity
U^+	Inner law velocity free stream U/v^*
u’	Root Mean Square (RMS) streamwise velocity fluctuation amplitude

u^+	Inner law velocity u/v^*
v'	Root Mean Square (RMS) cross-stream velocity fluctuation amplitude
v^*	Friction velocity $v^* = \sqrt{\frac{\tau_w}{\rho}}$
x	Streamwise spatial coordinate
x^*	x/δ
y	Cross-stream spatial coordinate
y^*	y/δ
y^+	Cross-stream inner law length scale $y^+ = \frac{yv^*}{\nu_w}$

Greek

α_0	Cross-stream velocity closure weighting; $2 < \alpha_0 < 4$
δ	Boundary layer thickness
δ^+	Boundary layer thickness inner law length scale $\delta^+ = \frac{\delta v^*}{\nu_w}$
ΔU	Pulsatile flow streamwise velocity amplitude
η	Pulsatile flow similarity independent variable: $\eta = y\sqrt{\frac{\omega}{\nu_{eff}}}$
κ	Von Karman constant κ=0.41
$\tilde{\kappa}$	Pulsatile flow modified Von Karman constant
ν	Kinematic viscosity
ω	Frequency
ω_0	Dimensionless frequency $\omega_0 = \frac{\omega\delta}{U}$
Φ	Power Spectral Density, i.e. spectra
ρ	Density
τ	Shear stress
τ	Auto-correlation time separation
θ	Momentum thickness

Subscripts/Superscripts

eff	Turbulent effective
inc	Incompressible
FT	Fully Turbulent
max	Maximum
os	Laminar-turbulent pressure "over-shoot"
pp	Pressure PSD
rms	Root Mean Square (RMS)
s	Steady
turb	Turbulent
t, tran	Transition
vehicle	Reentry vehicle
w	Wall
∞	Steady free-stream constant

I. INTRODUCTION

Vibratory loading of structures due to fluid turbulence excitation is a classical problem in fluid-structure interaction, Naudascher (1994) and Elishakoff, I (1983). Flow passage over structures, vehicle and other bodies, induce not only large scale, e.g. lift and drag effects, but small scale, unsteady, high frequency input that contribute to structural vibration. Physics based prediction of structure response requires concurrent estimation of high frequency (usually stochastic) velocity and pressure fluctuation behavior, i.e. turbulent fluctuations. It is possible to compute flow behavior directly from the Navier-Stokes equations without modeling in the very computationally intensive procedure termed "direct Numerical Simulation" DNS. Unfortunately, the number of floating point computations for DNS scales as Re^3 (Reynolds number cubed) Hoffman and Johnson (2006) suggesting that practical computations remain out of the reach for engineering simulation even for the current "petaflop" computers. Large Eddy Simulation, LES provides an alternative, but requires significant modeling at precisely the scale of interest You and Moin, P. (2007). Thus, modelling based efforts to predict turbulent velocity and pressure fluctuation are of great interest.

This paper discusses the formulation and approximate solution for wall bounded shear turbulent fluctuation quantities such as RMS velocity and pressure fluctuation using a sinusoidal pulsatile approach. This model follows as an extension to the turbulent laminar sublayer model developed by Sternberg (1962) to be valid for a fully turbulent flow domain. Though a single dominant individual mode of the turbulent flow fluctuation behavior is modeled, the approach yields plausible estimates for Root Mean Square (RMS) velocity and pressure fluctuations. RMS pressure fluctuations and spectra are of particular interest and are estimated via the pressure Poisson expression. Both RMS and Power Spectral Density (PSD), i.e. spectra are developed. Comparison with available measurements suggests reasonable agreement. An additional fluctuating quantity, i.e. RMS wall shear fluctuation is present in all wall bounded turbulent flows. However, to this point, this loading effect has been ignored as a loading input for reentry vehicles. To gain a sense of the relative importance of this input we the RMS wall shear fluctuation behavior. The models developed achieve reasonable agreement with measurement.

II. ANALYSIS/RESULTS

A. Pulsatile Flow/Velocity Fluctuation

The outer portion of the boundary layer is described by a fluctuation of the form $U'(x,t) = \Delta U \cos(kx + \omega t)(1 - \frac{y}{\delta})$ (Arpaci and Larson (1984)). This expression provides an estimate for the pressure fluctuation via:

$$\frac{\partial U'}{\partial t} + U'\frac{\partial U'}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} \tag{1}$$

The form of this expression suggests that $\frac{\partial U'}{\partial t} = O(\omega \Delta U)$ and $U'\frac{\partial U'}{\partial x} = O(\Delta U^2 (k + \frac{y}{\delta^2}\frac{d\delta}{dx}))$. We now propose that: $\Delta U \frac{y}{\delta^2}\frac{d\delta}{dx} << \Delta U k << \omega$ suggesting that we can initially ignore the convective term relative to the time dependent term.

We are interested in a near wall model and will construct expressions that local law of the wall profiles. Let's then consider the outer-law dominated unsteady flow:

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + \frac{\partial}{\partial y}\left(\nu_w \frac{\partial u'}{\partial y}\right)$$

$$\frac{\partial u'}{\partial t} - \frac{\partial U'}{\partial t} = \frac{\partial}{\partial y}\left(\nu_w \frac{\partial u'}{\partial y}\right) \tag{2}$$

where ν_w is the wall kinematic viscosity. Notice, that consistent with time averaged/steady models we have made the traditional approximation that convective terms are negligible very near the wall (log-layer).

To solve equation (2) we propose that a solution takes the form:

$$u' \approx \Delta U\left[\cos(kx + \omega t)(1 - \frac{y}{\delta}) - f(\eta)\cos(kx + \omega t - a\eta)\right] \quad ; \quad \eta = y\sqrt{\frac{\omega}{\nu_w}} \tag{3}$$

Substitution of this expression into equation (3) yields an expression in terms of $\cos(\omega t - a\eta)$ and $\sin(\omega t - a\eta)$. Demanding that both the cosine and sine expressions are individually satisfied yields two linear equations for f(η) of the form:

$$2a\frac{df}{d\eta} + f = 0 \rightarrow f = \exp(-\frac{1}{2a}\eta) \tag{4}$$

and

$$\frac{d^2 f}{d\eta^2} - a^2 f = 0 \rightarrow f = \exp(-a\eta) \tag{5}$$

Obviously, these two expressions can represent the same functional behavior by choosing "a" such that

$$\frac{1}{2a} = a \rightarrow a = \frac{\sqrt{2}}{2} \tag{6}$$

which yields $a = \frac{\sqrt{2}}{2}$. We can thus, estimate that the (unsteady) term is now:

$$\begin{aligned} u' &= \Delta U\left(\cos(kx + \omega t)(1 - \frac{y}{\delta}) - \exp(-\frac{\sqrt{2}}{2}\eta)\cos(kx + \omega t - \frac{\sqrt{2}}{2}\eta) \right) \\ &= \Delta U\left(\cos(kx + \omega t)(1 - \frac{y}{\delta}) - \exp(-\sqrt{\frac{\omega}{2\nu_w}}y)\cos(kx + \omega t - \sqrt{\frac{\omega}{2\nu_w}}y) \right) \end{aligned} \tag{7}$$

Equation (7) is an estimate for streamwise velocity fluctuation. It contains the unknown terms:

ΔU	=	fluctuation magnitude
ω	=	fluctuation frequency

The effective frequency is modeled as a velocity scale/length scale. Consider the expression:

$\omega \propto \frac{velocity}{length} = \omega_0 \frac{U}{\delta}$ and the wave number estimate: $k \approx \frac{k_0}{\delta}$ whereby we can write:

$$\begin{aligned} u' &= \Delta U\left(\cos\left(k_0\frac{x}{\delta} + \omega_0\frac{U}{\delta}t\right)(1 - \frac{y}{\delta}) - \exp(-\frac{\sqrt{2}}{2}y^+)\cos(k_0\frac{x}{\delta} + \omega_0\frac{U}{\delta}t - \frac{\sqrt{2}}{2}y^+) \right) \\ &= \Delta U\left(\cos\left(k_0\frac{x}{\delta} + \omega_0\frac{U}{\delta}t\right)(1 - \frac{y}{\delta}) - \exp(-\frac{\sqrt{2}}{2}y^+)\cos(k_0\frac{x}{\delta} + \omega_0\frac{U}{\delta}t - \frac{\sqrt{2}}{2}y^+) \right) \end{aligned} \tag{8}$$

Where $y^+ = \frac{v^* y}{\nu_w}$. The behavior of equation (8) mimics our concept of turbulent flow behavior since: $\frac{1}{2\pi}\int_0^{2\pi} u'(y^+, t)dt = 0$. It is convenient to place equation (8) in terms of outer variable as:

$$u' = \Delta U\left((\cos(k_0 \frac{x}{\delta} + \omega_0 \frac{U}{\delta} t)(1 - y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(k_0 \frac{x}{\delta} + \omega_0 \frac{U}{\delta} t - \frac{\sqrt{2}}{2}\delta^+ y^*) \right) \qquad (9)$$

Where $\delta^+ >> 1$ since using law of the wall: $\delta^+ = \exp(\kappa(U^+ - B)) = \exp(\kappa(\left(\frac{2}{C_f}\right)^{1/2} - B))$. Indeed for a "typical" value for the skin friction say C_f=0.002 we estimate that $\delta^+ \approx 400$.

With access to the streamwise fluctuation we need to estimate the cross-stream velocity, which follows from fluctuating continuity $\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$. Using continuity we can readily write:

$$\frac{\partial v'}{\partial y} = -\frac{\partial u'}{\partial x} \rightarrow v' = -\int \frac{\partial u'}{\partial x} dy \qquad (10)$$

While equation (8) is in closed form and relatively simple, formal application of equation (9) will result in in a rather unmanageable result. A simpler expression which retains essential overall physics is preferred. Let's examine the streamwise derivative $\frac{\partial u'}{\partial x}$ and the relative magnitudes of the streamwise variation as: $\frac{\partial u'}{\partial x} \propto O(\frac{\Delta U}{\delta}) >> O(\frac{\Delta U}{\delta}\frac{d\delta}{dx}) >> \frac{dv^*}{dx}$. Notice that the dominant term is associated with the spatial variation of the fluctuation via $\frac{d}{dx}(\cos(\frac{x}{\delta} + \omega_0 \frac{a_0}{\delta} t))$. An estimate for $\frac{\partial u'}{\partial x}$ can be written as: $\frac{\partial u'}{\partial x} \approx \frac{\Delta U_\infty}{\delta}\left((\sin(k_0 \frac{x}{\delta} + \omega_0 \frac{U}{\delta} t)(1 - y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\sin(k_0 \frac{x}{\delta} + \omega_0 \frac{U}{\delta} t - \frac{\sqrt{2}}{2}\delta^+ y^*) \right)$. In terms of the temporal fluctuating behavior the sine versus cosine functional behavior is rather arbitrary implying that an acceptable approximation for the streamwise derivative could be: $\frac{\partial u'}{\partial x} \propto \frac{u'}{\delta}$. We will discuss the implications of these models subsequently.

To compute the cross-stream fluctuation we need to then integrate $\frac{\partial u'}{\partial x}$ with respect to "y." While the integration can be formally computed, in keeping with the approximate nature of the streamwise velocity fluctuation we propose that a more traditional approximate scaling analysis may be useful. For example one can often (see Anderson et. al. 1984) interpret the cross stream velocity as: $v' = -\int \frac{\partial u'}{\partial x} dy \approx (\frac{u'}{\delta})\delta \propto u'$. A similar approximation (see White 2006) takes the form: $v' = -\int \frac{\partial u'}{\partial x} dy \approx \frac{u'}{\delta} y = y^* u'$. Which approach is appropriate? From an order of magnitude point of view they are both viable and hence an average expression might be useful. A useful average for these two models is geometric average (mean) such as $GM(X,Y) = (XY)^{1/2}$. As written, the geometric average implies equal weight of both X and Y, however the generalization: $GM(X,Y) = (X^a Y^b)^{1/(a+b)}$ is entirely appropriate as well. For example, one could readily pose a model of the form: $v' = (y^*)^{1/4} u'$ where we have utilized a weighted geometric average for the cross-stream integration length scale as: $(\delta^3 y^1)^{1/4}$.

$$\frac{u'}{\Delta U} = \left(\cos(k_0 \frac{x}{\delta} + \omega_0 \frac{U}{\delta} t)(1 - y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(k_0 \frac{x}{\delta} + \omega_0 \frac{U}{\delta} t + \frac{\sqrt{2}}{2}\delta^+ y^*) \right)$$
$$\frac{v'}{\Delta U} = (y^*)^{1/4} \left(\sin(k_0 \frac{x}{\delta} + \omega_0 \frac{U}{\delta} t)(1 - y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\sin(k_0 \frac{x}{\delta} + \omega_0 \frac{U}{\delta} t + \frac{\sqrt{2}}{2}\delta^+ y^*) \right) \quad (11)$$

We note that in terms of the temporal fluctuating behavior the sine versus cosine functional behavior is rather arbitrary for u' and v' individually, but will have important consequences as we look at cross-correlation behavior, i.e. terms like $\langle u'v' \rangle \equiv \frac{1}{T}\int_0^T u'v' dt$.

Obviously both the streamwise fluctuation u' and the cross-stream fluctuations v' are functions described by $\cos(k_0 \frac{x}{\delta} + \omega_0 \frac{a_0}{\delta} t)$ and $\sin(k_0 \frac{x}{\delta} + \omega_0 \frac{a_0}{\delta} t)$ This functional behavior roughly describes spatial and temporal fluctuations associated with near velocity fluctuations. The wave number variation is important as associated with the local continuity expression, but it is of less value in determining global fluctuation behavior. Indeed we can replace the streamwise term $\frac{x}{\delta}$ by $\frac{Ut}{\delta}$ and the cosine terms by $\cos((\omega_0 + k_0)\frac{U}{\delta} t)$ and $\sin((\omega_0 + k_0)\frac{U}{\delta} t)$ which effectively makes the turbulent fluctuation strictly time dependent.

B. Root Mean Square RMS Velocity Fluctuations

There is value in obtaining a root-mean-square (RMS) value for both u' and v'. By observing that the temporal behavior associated with these models is sinusoidal, the RMS value is readily obtained since for a function like: $g(t) = g_0 \cos(2\pi f t + a_0)$ the RMS value is computed as:

$$g_{rms} = g_0 \sqrt{\int_0^1 \cos^2(2\pi f t + a_0) dt} = \frac{\sqrt{2}}{2} g_0 \tag{12}$$

Notice that this result is insensitive to both the frequency and the phase shift a_0. Applying equation (12) we can write:

$$\begin{aligned} \frac{u'_{rms}}{\Delta U} &= \frac{\sqrt{2}}{2}\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(\frac{\sqrt{2}}{2}\delta^+ y^*)\right) \\ \frac{v'_{rms}}{\Delta U} &= \frac{\sqrt{2}}{2}(y^*)^{1/4}\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\sin(\frac{\sqrt{2}}{2}\delta^+ y^*)\right) \end{aligned} \tag{13}$$

We need to estimate the velocity scale ΔU. A plausible way in which to proceed is to consider the Reynolds shear stress $-\langle u'v' \rangle|$. We can readily compute $\langle u'v' \rangle \equiv \frac{1}{T}\int_0^T u'v'dt$. However, direct application of expressions in equation (11) yields (perhaps unsurprisingly) $\langle u'v' \rangle \equiv \frac{1}{T}\int_0^T u'v'dt = 0$ since the $\cos((\omega_0 + k_0)\frac{U}{\delta}t)$ and $\sin((\omega_0 + k_0)\frac{U}{\delta}t)$ representations are orthogonal with

$$\left\langle \cos((\omega_0 + k_0)\frac{U}{\delta}t)\sin((\omega\ + k_0)\frac{U}{\delta}t) \right\rangle = 0$$

Clearly the choice associated with representation of the sinusoidal portion of the model does indeed make a difference. Previously we noted that representation of v' as $v' \propto u' y^{*1/4}$ was, from approximate integration point of view as plausible as equation (11). This being the case one could estimate the magnitude of the Reynolds stress as: $\langle u'v' \rangle \propto u'_{rms}\, v'_{rms}$ such that:

$$\frac{\langle u'v' \rangle}{\Delta U^2} \propto \frac{1}{2}\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(\frac{\sqrt{2}}{2}\delta^+ y^*)\right)^2 (y^*)^{1/4} \tag{14}$$

with: $\left\langle \cos^2((\omega_0 + k_0)\frac{U}{\delta}t) \right\rangle = \frac{1}{2}$. Certainly, the representation of u'v' strictly based upon $\left\langle \cos((\omega_0 + k_0)\frac{U}{\delta}t)\sin((\omega \ + k_0)\frac{U}{\delta}t) \right\rangle$ provides no information, one could use a simple arithmetic average of approaches with would modify the weighting constant to be ¼ rather than ½. Retaining this degree of generality by defining the weighting constant (say) α_0 where $1< \alpha_0<2$

Equation (14) then can be written:

$$\langle u'v' \rangle \propto \frac{\Delta U^2}{2\alpha_0}\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(\frac{\sqrt{2}}{2}\delta^+ y^*) \right)^2 (y^*)^{1/4} \tag{15}$$

Examining the maximum value for the function $\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(\frac{\sqrt{2}}{2}\delta^+ y^*) \right)^2 (y^*)^{1/4} \bigg|_{\max} = 0.46 \approx \frac{1}{2}$. Demanding that the maximum Reynolds shears stress equal the wall value we have: $v^{*2} = \frac{\Delta U^2}{2\alpha_0}\left(\frac{1}{2}\right) \rightarrow \Delta U = 2\sqrt{\alpha_0}v^* = \sqrt{2\alpha_0}C_f^{1/2}U$.

The root mean square fluctuations can then be written as:

$$\begin{aligned} \frac{u'_{rms}}{U} &= \sqrt{\alpha_0}C_f^{1/2}\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(\frac{\sqrt{2}}{2}\delta^+ y^*) \right) \\ \frac{v'_{rms}}{U} &= \sqrt{\alpha_0}C_f^{1/2}(y^*)^{1/4}\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\sin(\frac{\sqrt{2}}{2}\delta^+ y^*) \right) \end{aligned} \tag{16}$$

Or in terms of inner variables

$$\begin{aligned} \frac{u'_{rms}}{v^*} &= \sqrt{2\alpha_0}\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(\frac{\sqrt{2}}{2}\delta^+ y^*) \right) \\ \frac{v'_{rms}}{v^*} &= \sqrt{2\alpha_0}(y^*)^{1/4}\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\sin(\frac{\sqrt{2}}{2}\delta^+ y^*) \right) \end{aligned} \tag{17}$$

Using equation (16) we can plot estimates for the streamwise velocity fluctuation and the cross-stream fluctuation. We use the same parameters $\delta^+ \approx 400$, Re_x= 1E7 and $C_f \approx 0.002$ gives:

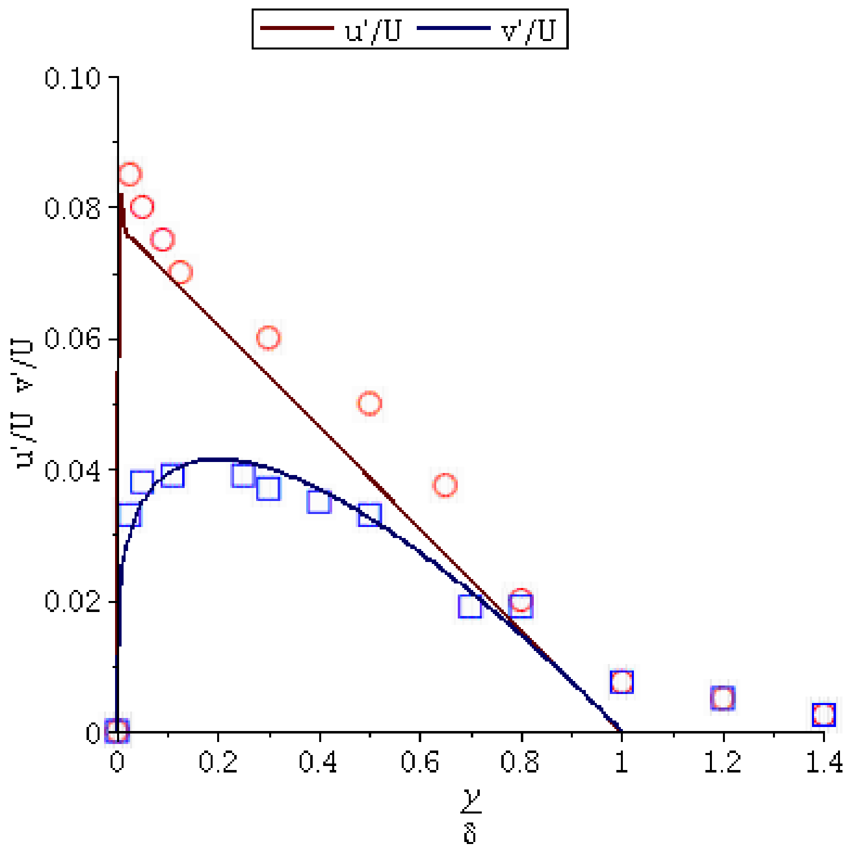

Figure 1.Comparison between simplified estimates for RMS velocity fluctuation u'/U and v'/U and measurement: Klebanoff (1955) using the "weighted" geometric average model for v'/U i.e. suggesting improved agreement with data. Additionally, α_0=2.

An estimate for the turbulent Reynolds wall shear stress $\langle u'v'\rangle \equiv \frac{1}{T}\int_0^T u'v'dt$ is an essential part of our model. Using the preceding estimates we readily obtain an estimate as:

$$\frac{\langle u'v'\rangle}{U^2} = 2\alpha_0 C_f\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(\frac{\sqrt{2}}{2}\delta^+ y^*)\right)^2 (y^*)^{1/4}$$

(18)

$$\frac{\langle u'v'\rangle}{v^{*2}} = 2\left((1-y^*) - \exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos(\frac{\sqrt{2}}{2}\delta^+ y^*)\right)^2 (y^*)^{1/4}$$

The magnitude by definition follows from the friction velocity, but there is value in plotting against the Klebanoff data set who plots $20\frac{\langle u'v' \rangle}{U^2}$. Comparing equation (18) with the Klebanoff data, i.e. $20\frac{\langle u'v' \rangle}{U^2}$ gives figure 3:

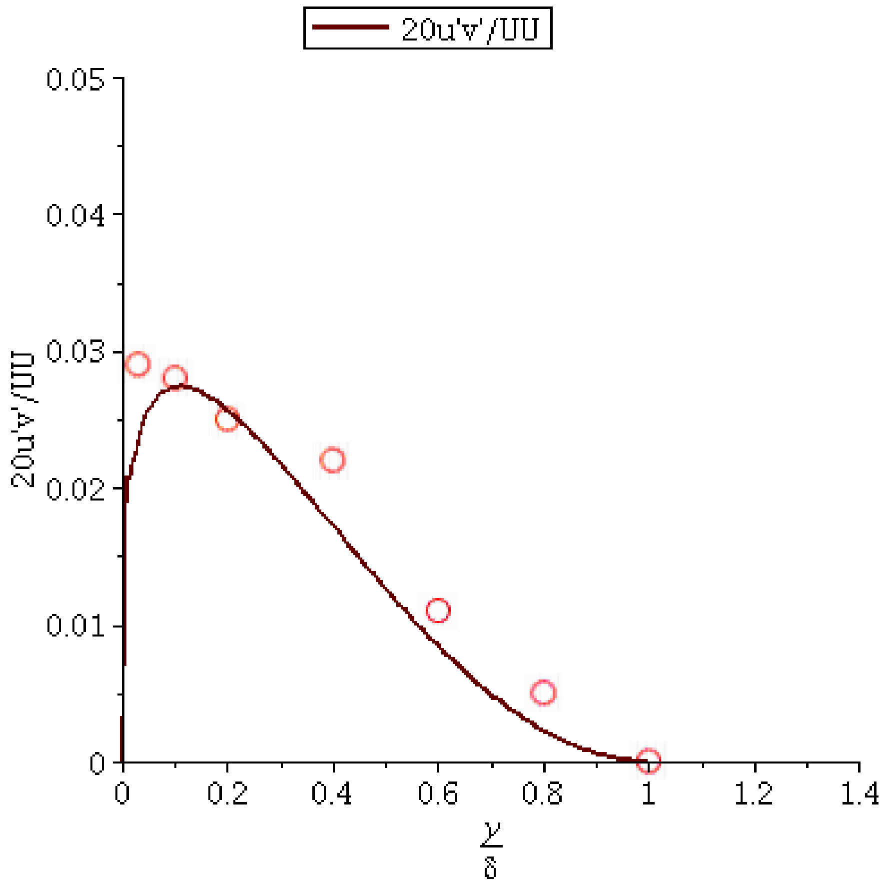

Figure 2. Comparison between simplified estimate for turbulent shear stress and measurement: Klebanoff (1955) using the "weighted" geometric average model for v'/U

Let's summarize what we have (and have not) accomplished. We have achieved simplified estimates for the functional behavior associated with Using a simplified extension of a time unsteady sinusoidal periodic/pulsatile flow analysis we have estimated expressions for fluctuating velocity with attendant functional behavior. The results provide reasonable agreement with classical data sets. We emphasize, that the modeling is rather crude and the arguments utilized are little more than order of magnitude scaling. Nonetheless, for some applications they can provide useful insight in to flow behavior.

C. Pressure Fluctuation

It is often necessary to estimate the RMS wall pressure fluctuation p'_{rms}. We consider the Poisson equation pressure fluctuations (Lilley 1963) given by:

$$\frac{1}{a_0^2}\frac{\partial^2 p'_{rms}}{\partial t^2}-\frac{\partial^2 p'_{rms}}{\partial x^2}-\frac{\partial^2 p'_{rms}}{\partial y^2}=2\rho\left|\left(\frac{\partial v'_{rms}}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)\right| \tag{19}$$

A reasonable approximation (consistent with the Taylor hypothesis (Blake 1986)) is to conclude that $t=\frac{x}{U}$ such that equation (19) can be written as: $(M^2-1)\frac{\partial^2 p'_{rms}}{\partial x^2}-\frac{\partial^2 p'_{rms}}{\partial y^2}=2\rho\left|\left(\frac{\partial v'_{rms}}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)\right|$

Notice that we have included the dominant mean shear-turbulence term only and neglected the turbulent-turbulent term. The absolute value of the source term is employed here with the idea that $p'\equiv p'_{rms}$. The pressure Poisson equation follows from the highly simplified mean flow model with $u=u(y)$. This model follows from the time averaged flow with streamwise variation ignored. Notice that this is a rather more drastic approximation that utilized to derive the RMS models. Indeed, though we ignore, $\frac{\partial u}{\partial x}$ behavior in the initial derivation we will generalize the final result to include streamwise variation. Though a linear Poisson equation (19) is formally solvable in closed form the result (in terms of Green's functions or eigenfunction expansions) Rather than utilize these approaches we consider a simplified approach.

Our first task is to model the source term $2\rho\left(\frac{\partial v'_{rms}}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)$ Using the log law we can estimate the mean flow derivative as: $\frac{\partial u}{\partial y}=\frac{v^*}{\kappa}\frac{1}{y}$ while the streamwise fluctuation term is: $\frac{\partial v'_{rms}}{\partial x}\propto\frac{dv^*}{dx}(1-y^*)y^{*1/4}$. Combining terms yields: $\frac{\partial u}{\partial y}\frac{\partial v'_{rms}}{\partial x}\approx\frac{\sqrt{2\alpha_0}}{\kappa\delta}(1-y^*)y^{*-3/4}\frac{d}{dx}\left(v^{*2}\right)$. It will be convenient to non-dimensionalize using the Reynolds number so as to write:

$$(M^2-1)\frac{\partial^2 p'_{rms}}{\partial x^2}-\frac{1}{\delta^2}\frac{\partial^2 p'_{rms}}{\partial y^{*2}}=\rho\frac{\sqrt{2\alpha_0}}{\kappa\delta}(1-y^*)y^{*-3/4}\frac{d}{dx}\left(v^{*2}\right) \tag{20}$$

with boundary conditions: $\left.\frac{\partial p'_{rms}}{\partial y^*}\right|_0 = p'_{rms}(1) = 0$. We can roughly approximate: $\frac{d}{dx}\left(v^{*2}\right) \approx c_0 \frac{v^{*2}}{x}$ (we discuss the appropriate constant) and further (assuming slow variation of δ) define the new variable: $x^* = \frac{x}{\delta}$ whereby equation (20) becomes:

$$\frac{\partial^2 p'_{rms}}{\partial x^{*2}} - \frac{\partial^2 p'_{rms}}{\partial y^{*2}} = \rho c_0 \frac{\sqrt{2\alpha_0}}{\kappa}(1-y^*)y^{*-3/4}\frac{v^{*2}}{x^*} \tag{21}$$

The proposed solution approach for equation (21) is based upon a simple traditional Galerkin approach. (Fletcher (1984)). We propose a product solution of the form: $p'_{rms}(x^*, y^*) = f(x^*)g(y^*)$. Then following the Galerkin method (Rayleigh-Ritz etc.) we propose a solution expression for $g(y^*)$ as $g(y^*) = \cos\left(\frac{\pi}{2}y^*\right)$ which, of course, is an eigenfunction for the homogeneous portion of the equation and satisfies the BC $\left.\frac{\partial p'_{rms}}{\partial y^*}\right|_0 = p'_{rms}(1) = 0$. We can then substitute the solution expression into the governing equation to give:

$$\frac{1}{2}\left((M^2-1)\frac{d^2 f}{dx^{*2}} + \frac{\pi^2}{4}(\omega_0 + k_0)^2 f\right) = \rho c_0 \frac{\sqrt{2\alpha_0}}{\kappa}\frac{v^{*2}}{x^*}\int_0^1 (1-y^*)y^{*-3/4}\cos\left(\frac{\pi}{2}y^*\right)dy^* \tag{22}$$

Where we gave used $\int_0^1 \cos^2\left(\frac{\pi}{2}y^*\right)dy^* = \frac{1}{2}$. The RHS integral (though complex) can be evaluated in closed form to give: $\int_0^1 (1-y^*)y^{*-3/4}\cos\left(\frac{\pi}{2}y^*\right)dy^* \approx 3$. Equation (21) then represents the ODE for f(x) as:

$$(M^2-1)\frac{d^2 f}{dx^{*2}} + \frac{\pi^2}{4}(\omega_0 + k_0)^2 f = \rho c_0 6\frac{\sqrt{2}}{\kappa}\frac{v^{*2}}{x^*} \tag{23}$$

It is convenient to approximate the frequency constant ω and wave number constants k as $\omega + k = \sqrt{M^2 - 1}$ which implies that the flow problems of interest are supersonic. We define: $\beta \equiv \sqrt{M^2-1}$

Equation (23) is solvable in terms of a sum composed of homogeneous and particular solutions as:

$$f(x^*) = \frac{\rho c_0}{\beta^2}\frac{\sqrt{2}}{\kappa}6v^{*2}\left(\cos\left(\frac{\pi}{2}x^*\right) + \frac{2}{\pi}(Ci\left(\frac{\pi}{2}x^*\right)\sin\left(\frac{\pi}{2}x^*\right) - Si\left(\frac{\pi}{2}x^*\right)\cos\left(\frac{\pi}{2}x^*\right))\right) \quad (24)$$

The homogeneous solution has been chosen in combination with the particular solution such that $f(x^*)$ is non oscillatory. It is worth noting that the potentially singular behavior associated with equation (23) can be mitigated by modeling the RHS in terms of:

$$\frac{d^2 f}{dx^{*2}} + \frac{\pi^2}{4}f = \rho\frac{c_0}{\beta^2}6\frac{\sqrt{2}}{\kappa}\frac{v^{*2}}{(x^* + 1)} \quad (25)$$

Integrating this expression and requiring non-oscillatory behavior yields:

$$\frac{f(x^*)}{\rho\frac{c_0}{\beta^2}\frac{\sqrt{2}}{\kappa}6} = v^{*2}\left(\frac{2}{\pi}(Si\left(\frac{\pi}{2}(x^* + 1)\right)\sin\left(\frac{\pi}{2}x^*\right) + Ci\left(\frac{\pi}{2}(x^* + 1)\right)\cos\left(\frac{\pi}{2}x^*\right)) - \sin\left(\frac{\pi}{2}x^*\right)\right) \quad (26)$$

which yields similar behavior to equation (24).

With the solution for $f(x^*)$ we can estimate $p'_{rms}(x^*, y^*)$. The x^* is arbitrary (bounded between $0 < x^* < \infty$); for $x^* >> 1$ the source term in equation (21) is zero and $p'_{rms}(x^*, y^*) \to 0$. The maximum value for $p'_{rms}(x^*, y^*)$ is $p'_{rms}(0,0)$ and is written as: $p'_{rms} \approx 6\frac{c_0}{\beta^2}\frac{\sqrt{2\alpha_0}}{\kappa}\rho v^{*2}$

To be of further use, we need to estimate the "c$_0$" term $\frac{d}{dx}\left(v^{*2}\right) \approx c_0\frac{v^{*2}}{x}$. The definition of the friction velocity is such that $v^{*2} = \frac{1}{2}C_f$. If we model $C_f = A\,\mathrm{Re}_x^{-a}$ then $\left|\frac{d}{d\,\mathrm{Re}_x}(\frac{1}{2}C_f)\right| = a\frac{1}{2}\frac{C_f}{\mathrm{Re}_x} = \frac{av^{*2}}{\mathrm{Re}_x}$.

Power-law expressions for the skin friction suggest that: 1/7<a<1/5 which implies that

$$3\rho v^{*2} < p' < 4.1\rho v^{*2} \quad (27)$$

For α_0=1, while for α_0=2 we have:

$$4.2\rho v^{*2} < p' < 6.0\rho v^{*2} \quad (28)$$

DeChant (2015) provides additional discussion. Note, that here we have assumed that $\beta = \sqrt{M^2 - 1} \approx 1$ implying that M=√2

The success of the model in providing reasonable estimates for the wall RMS pressure fluctuation suggests that the model may provide reasonable estimates for the frequency spectrum or power spectral density. We start with the pressure fluctuation Poisson equation as:

$$(M^2-1)\frac{\partial^2 p'}{\partial x^2}-\frac{\partial^2 p'}{\partial y^2}=2\rho\left(\frac{\partial v'}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right) \tag{30}$$

Notice that the pressure fluctuation directly "inherits" the temporal behavior from the velocity fluctuation as:

$$\frac{v'}{\Delta U}=(y^*)^{1/4}\left(\cos((k_0+\omega_0)\frac{x}{\delta})(1-y^*)-\exp(-\frac{\sqrt{2}}{2}\delta^+ y^*)\cos((k_0+\omega_0)\frac{x}{\delta}+\frac{\sqrt{2}}{2}\delta^+ y^*)\right) \tag{31}$$

where we will utilize $v' \propto u' y^{*1/4}$. Since we were able to successfully estimate p'$_{rms}$ we focus on the spectral functional behavior for the Φ_{pp} and compute the magnitude separately. Indeed, to first order one can propose a product estimate for the behavior for p'(x,y,t) as:

$$p' \propto \cos((\omega_0+k_0)\frac{U}{\delta}t)\rho v^{*2} g(y^*) f(x^*)=\cos((\omega_0+k_0)\frac{U}{\delta}t)\rho v^{*2} f(x^*)\cos(\frac{\pi}{2}y^*) \tag{32}$$

where $f(x^*)$ follows from either equation (24) or equation (26).

To estimate the frequency spectrum we consider the Fourier transform pair (DeChant (2014)):

$$\begin{aligned}\Phi_{pp}(\omega)&=\frac{1}{2\pi}\int_0^\infty R(\tau)\cos(\omega t)d\tau\\ R_{pp}(\tau)&=\int_0^\infty \Phi_{pp}(\omega)\cos(\omega t)d\tau\end{aligned} \tag{33}$$

Obviously to utilize the transform pair we must have an estimate for the correlation function R(τ). The autocorrelation R(τ) follows from $R_{pp}(\tau)=\frac{1}{T}\int_0^T p'(x,y,t)p'(x,y,t+\tau)dt$. The product form of equation (32) and the assignment for the time period as: $T\equiv\left(\omega_0+k_0)\frac{U}{\delta}\right)^{-1}$ yields:

$$R_{pp}(\tau)\propto\cos((\omega_0+k_0)\tau^*)\rho v^{*2} g(y^*) f(x^*) \quad ; \quad t^*=\frac{Ut}{\delta} \tag{34}$$

Using the transform pair i.e. equation (32) we can then estimate the frequency spectrum.

Note, however, that from Taylor's frozen turbulence hypothesis that streamwise convective behavior is equivalent to temporal behavior via: $Ut=x\to t^*=\frac{Ut}{\delta}=\frac{x}{\delta}$ hence we need to include streamwise behavior in our estimate for Φ_{pp}. We emphasize that we can map between dimensionless time and dimensionless space as needed. The streamwise behavior for our estimate to the pressure fluctuation is given by equation (24) and takes the form: $\left(\cos\left(\frac{\pi}{2}x^*\right)+\frac{2}{\pi}(Ci\left(\frac{\pi}{2}x^*\right)\sin\left(\frac{\pi}{2}x^*\right)-Si\left(\frac{\pi}{2}x^*\right)\cos\left(\frac{\pi}{2}x^*\right))\right)$. This expression in the current form is unnecessarily complex and can be approximated by:

$$\left(\cos\left(\frac{\pi}{2}x^*\right)+\frac{2}{\pi}(Ci\left(\frac{\pi}{2}x^*\right)\sin\left(\frac{\pi}{2}x^*\right)-Si\left(\frac{\pi}{2}x^*\right)\cos\left(\frac{\pi}{2}x^*\right))\right)\approx\frac{1}{(1+x^*)^{3/2}}\approx\frac{1}{(1+x^*)} \qquad (35)$$

implying that $R_{pp}(\tau)\propto R_{pp}(x^*)=\frac{\cos((\omega_0+k_0)x^*)}{(1+x^*)^{3/2}}\approx\frac{\cos((\omega_0+k_0)x^*)}{(1+x^*)}$. By way of comparison, a “traditional” correlation function expression is $R_{pp}(x^*)=\exp(-cx^*)$ with c=1, where the rate of change is chosen to match equation (35) to first order. Note, that the cosine term in the correlation expression can be associated with the traditional modeling approach for narrow band correlation functions (Bakewell (1964) and Clinch (1969)) models. Broadband models simply remove the cosine term but retain the decay behavior. Obviously, for the $(\omega_0+k_0)<<1$ the broadband and narrow band results are coincident. Though certainly $(\omega_0+k_0)=0$ will honor the broadband result, a zero frequency limit is not meaningful. Nonetheless, $(\omega_0+k_0)=O(1/10)$ will yield results that mimic the broadband limit. Moreover, the classical Corcos result for $R(\xi)=R(\xi,0,0)\propto\int_0^\infty\phi(\omega)A(\frac{\omega\xi}{U})\cos(\frac{\omega\xi}{U})d\omega$ is often modeled as: $A(\frac{\omega\xi}{U})=\exp(-0.11\xi)$ suggesting that $(\omega_0+k_0)\approx 0.11$.

The estimate for the function behavior associated with the narrow spectral density (band center $(\omega_0+k_0)=1$) and broadband expressions can be written using:

$$\begin{aligned}\Phi_{pp\_n}(\omega^*)&\propto\int_0^\infty\frac{\cos((\omega_0+k_0)x^*)}{(1+x^*)^{3/2}}\cos(\omega x^*)dx^*\\ \Phi_{pp\_b}(\omega^*)&\propto\int_0^\infty\frac{1}{(1+x^*)^{3/2}}\cos(\omega x^*)dx^*\end{aligned} \qquad (36)$$

Or

$$
\begin{aligned}
\Phi_{pp\_n}(\omega^*) &\propto \int_0^\infty \frac{\cos((\omega_0 + k_0)x^*)}{(1+x^*)} \cos(\omega^* x^*) dx^* \\
\Phi_{pp\_b}(\omega^*) &\propto \int_0^\infty \frac{1}{(1+x^*)} \cos(\omega^* x^*) dx
\end{aligned}
\tag{37}
$$

The result for both integrals is available in closed form but is complex and is most useful to simply plot the results. There is value in comparing the pressure fluctuation spectra as estimated by equation (36) or (37) to other approaches. As a starting point, we note, that pressure fluctuation source term $2\rho\left|\left(\frac{\partial v'}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)\right|$ contains unsteady information strictly through the $\frac{\partial v'}{\partial x}$ term. Thus, one could simply approximate the pressure fluctuation spectrum as a scaled function of velocity fluctuation spectrum. Though normally used for continuous wind gust encounters, either the Dryden $\Phi_{pp}(\omega) \propto \frac{1+12\omega^2}{\left(1+4\omega^2\right)^2}$ or the Von Karman $\Phi_{pp}(\omega) \propto \frac{1+\frac{8}{3}(2.7\omega)^2}{\left(1+(2.7\omega)^2\right)^{11/6}}$ velocity fluctuation spectra could be considered.

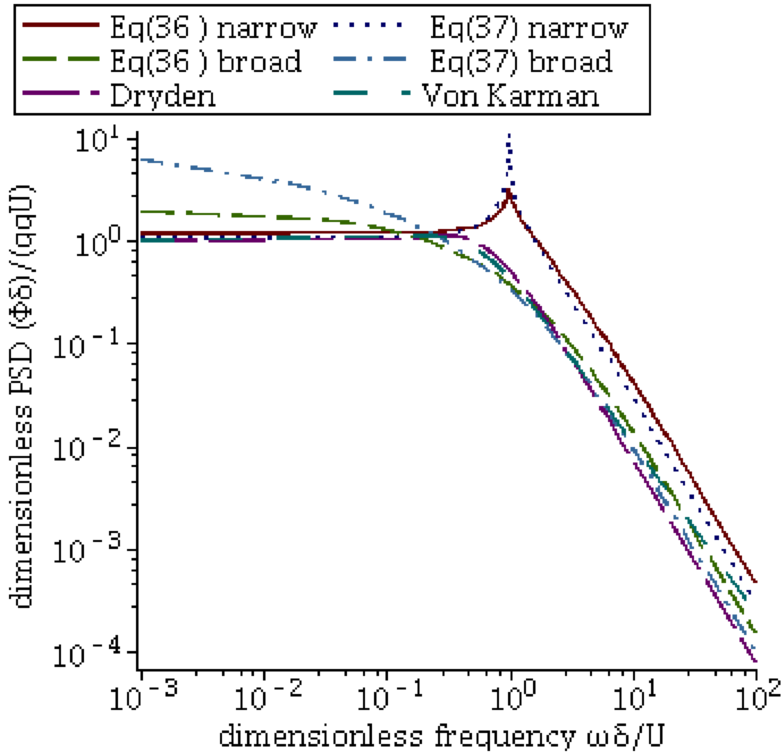

Figure 3. Pressure fluctuation root mean spectrum (PSD) functional behavior computed using equation (36) , equation (37) and spectra based upon the Dryden and Von Karman continuous wind gust distribution.

The narrow band model for ω_0=1 is characterized by a low frequency i.e. $\omega^* = \dfrac{\omega\delta}{U} < 1$ constant behavior, a local peak for $\omega^* = 1$ and decay for $\omega^* > 1$. The peak at $\omega^* = 1$ corresponds to the influence of the sinusoidal term in the transform kernel since $\int_0^\infty \cos(x^*)\cos(\omega^* x^*)dx^* = \delta(\omega^* - 1)$. The high frequency decay follows as $\Phi_{pp}(\omega) \propto \omega^{*-2}$. A lower frequency band center, i.e. (ω_0+k_0)=1/10 problem is similar with constant behavior for $\omega^* = \dfrac{\omega\delta}{U} < \dfrac{1}{10}$ a weak pulse for $\omega^* = \dfrac{1}{10}$ and decay for higher frequencies. The particular meaning associated this value for $\omega^* = \dfrac{\omega\delta}{U}$ follows from the fact that it is common to use a dimensionless frequency of the form $\delta_{disp} \approx \dfrac{1}{8}\delta \rightarrow \dfrac{\omega\delta_{disp}}{U} = \dfrac{1}{8}$.

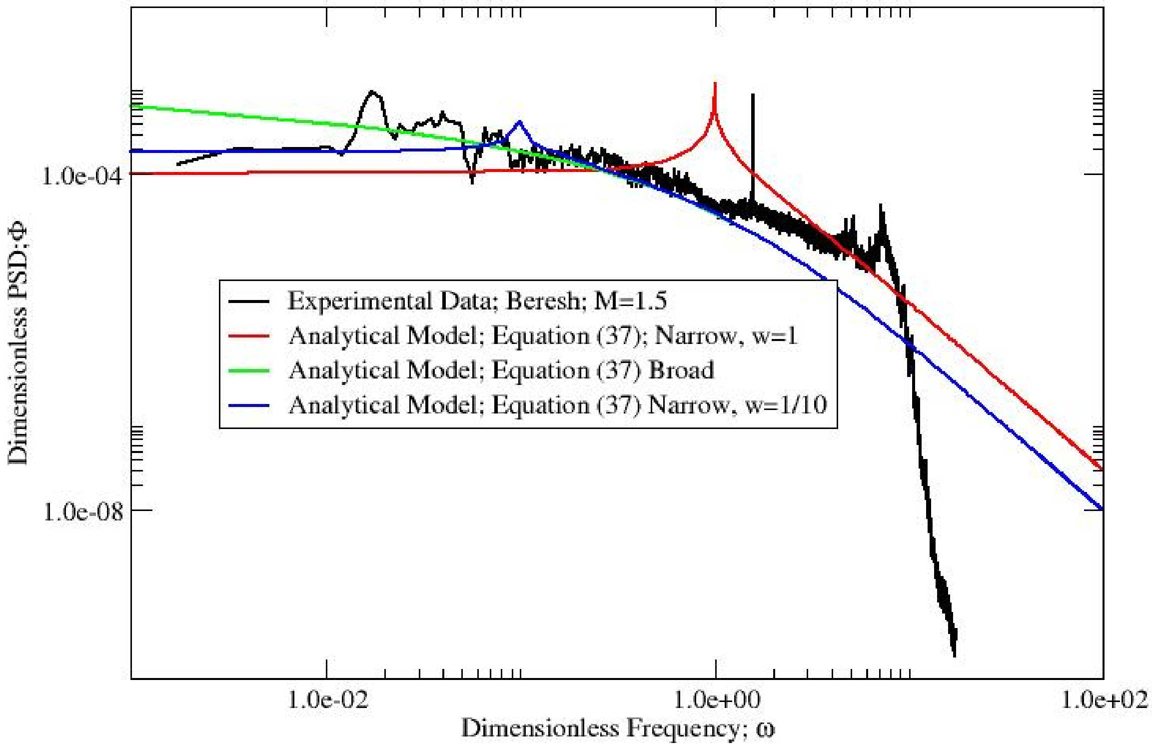

Figure 4. Pressure fluctuation root mean spectrum (PSD) functional behavior computed using equation (36) and equation (37) compared to M=1.5 data set Beresh (2011) (See DeChant 2015).

Of course, there is little value in estimating spectral behavior without comparison to data. In figure 5 we compare our spectral estimate to the data of Beresh et. al. (2011). The model suggests reasonable agreement for low frequency behavior and (perhaps) adequate agreement for a portion of the spectrum with $\omega^* > 1$. The high frequency behavior is not adequately modeled since the

D. Wall Shear Fluctuation

Estimates for the root mean square of the fluctuating turbulent wall shear are of particular interest. We will examine two approaches: (1) estimation using Reynolds stress definition and (2) a local wall analog model described by Alfredsson et. al. (1988). We can define the as: $u'v' - \langle u'v' \rangle$. Using the preceding models let's examine estimates for RMS for this quantity as:

$$u'v'\big|_{rms} \equiv \left\langle u'^2 \, v'^2 \right\rangle - \left\langle u'v' \right\rangle \tag{38}$$

and $\langle \cdot \rangle \equiv \dfrac{1}{T}\int_0^T \cdot \, dt$. We then use the (approximate) preceding temporal expressions:

$$\frac{u'}{\Delta U} \approx \left(\cos(\omega_0 (\frac{U}{\delta} + \frac{v^{*2}}{\nu_w})t)(1-y^*) \right)$$
$$\frac{v'}{\Delta U} \approx \frac{(y^*)^{1/4}}{\alpha_0}(1-y^*)\left(\cos(\omega_0 (\frac{U}{\delta} + \frac{v^{*2}}{\nu_w})t)) \right) \tag{39}$$

Where α_0=2 implying the simple weighted average. The term Reynolds stress term $\langle u'v' \rangle$ follows as:

$$\langle u'v' \rangle \approx \frac{\Delta U^2}{2\alpha_0}\left((1-y^*)\right)^2 (y^*)^{1/4} \tag{40}$$

Applying the RMS definition to the fluctuating Reynolds stress $u'v' - \langle u'v' \rangle$ we have:

$$\frac{\left(u'v'|_{rms}\right)^2}{\frac{\Delta U^4}{\alpha_0^2}(y^*)^{1/2}(1-y^*)^2} = \left\langle \left(\cos^2(wt) - \frac{1}{2} \right)^2 \right\rangle \tag{41}$$

Where $w \equiv \omega_0 \left(\frac{U}{\delta} + \frac{v^{*2}}{\nu_w} \right)$. Over a periodic interval $\left\langle \left(\cos^2(wt) - \frac{1}{2} \right)^2 \right\rangle = \frac{1}{8}$ implying that we can estimate:

$$\left(u'v'|_{rms}\right) \approx \frac{\Delta U^2}{2\sqrt{2}\alpha_0}(y^*)^{1/4}(1-y^*) = \sqrt{2} v^{*2} (y^*)^{1/4}(1-y^*) \tag{42}$$

Where we have used: $\Delta U = 2\sqrt{\alpha_0} v^*$. Now the maximum value associated with $(y^*)^{1/4}(1-y^*)\big|_{\max} \approx 0.53 \approx \frac{1}{2}$ which suggests that we can estimate the RMS wall shear flux is $\left(u'v'|_{rms}\right) \approx \frac{\sqrt{2}}{2} v^{*2}$

The local wall analog model described by Alfredsson et. al. (1988) follows from laminar sublayer modeling estimates as:

$$\left(u'v'|_{rms\_wall}\right) \approx \lim_{y \to 0} \frac{u'_{rms}}{u} \tag{42}$$

where both the RMS velocity fluctuation and the local mean velocity must be estimated in the viscous sub-layer. Using equation (13) we estimate the near wall streamwise RMS velocity u'_{rms} as:

$$u'_{rms} = \frac{\sqrt{2}}{2}\Delta U \left(\frac{\sqrt{2}}{2}\delta^+ - 1 \right) y^* \approx \frac{1}{2}\Delta U \frac{yv^*}{\nu_w} \tag{43}$$

Since $\delta^{+^{-1}} << 1$, while the mean velocity is simply:

$$u^+ = y^+ \rightarrow u = v^{*2} \frac{y}{\nu_w} \tag{44}$$

Using these two velocity estimates and $\Delta U = 2\sqrt{\alpha_0} v^*$ we write:

$$\left(u'v'\big|_{rms\_wall} \right) \approx \sqrt{\alpha_0} v^{*2} \tag{45}$$

Which with α_0 suggests that $\left(u'v'\big|_{rms\_wall} \right) \approx \sqrt{2} v^{*2}$.

Experimental data for RMS wall shear datasets follows from Mathis et. al. provides a Reynolds number dependent expression as:

$$\tau'_{w\_rms} = \left(0.298 + 0.018 \ln(\mathrm{Re}_\tau)\right)\rho v^{*2} \tag{46}$$

We can rewrite Re_τ as $\mathrm{Re}_\tau = \frac{v^*\delta}{\nu_w} = \frac{\sqrt{2}}{2} C_f^{1/2} \mathrm{Re}_\delta \approx \frac{\sqrt{2}}{2} C_f^{1/2} (0.16) \mathrm{Re}_x^{6/7}$. For Re_x=1E8 and C_f=0.005 we suggest that $\tau'_{w\_rms} \approx 0.48$. Obviously, comparison with $\left(u'v'\big|_{rms} \right) \approx \frac{\sqrt{2}}{2} v^{*2}$ is rather less promising. We remark, however, that the estimates are correct to an order of magnitude.

III. CONCLUSIONS

This report provided the formulation and approximate solution for wall bounded shear turbulent fluctuation quantities such as RMS velocity and pressure fluctuation using a sinusoidal pulsatile approach which is an extension to the turbulent laminar sublayer model developed by Sternberg (1962) for a fully turbulent flow domain. Though limited to a single dominant individual mode of the turbulent flow fluctuation behavior is modeled, the approach provides plausible estimates for Root Mean Square (RMS) velocity and pressure fluctuations. Focus was placed on pressure fluctuations and spectra which were estimated via the pressure Poisson expression. Comparison with available measurements suggests moderate agreement for low frequency but poor agreement for the high frequency portion of the spectrum. An additional fluctuating quantity, i.e. RMS wall shear fluctuation which is present in all wall bounded turbulent flows was modeled as well. The model for RMS wall shear fluctuation achieved reasonable agreement with measurement.

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